

Resonant production of the sterile neutrino dark matter and fine-tunings in the ν MSM

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The generation of lepton asymmetry below the electroweak scale has a considerable impact on production of dark matter sterile neutrinos. Oscillations or decays of the heavier sterile neutrinos in the neutrino minimal standard model can give rise to the requisite lepton asymmetry, provided the masses of the heavier neutrinos are sufficiently degenerate. We study the renormalization group evolution of the mass difference of these singlet fermions to understand the degree of necessary fine-tuning. We construct an example of the model that can lead to a technically natural realization of this low-energy degeneracy.

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I. INTRODUCTION

The neutrino minimal standard model (ν MSM) is a renormalizable extension of the standard model (SM) by three singlet Majorana fermions with masses below the electroweak scale. As has been elaborated in Refs. [1–4] (for a review see [5]), this model enables us to solve, by economic means, four observational problems of the SM. Owing to the low-energy see-saw mechanism the ν MSM leads to nonzero masses of the active neutrino flavors and thus to neutrino oscillations. The model provides a candidate for dark matter (DM) particle in form of a sterile neutrino [6–9] in the mass range of 1 – 50 keV (see [5] for a review). The coherent oscillations of the heavier neutral leptons insure the generation of baryon asymmetry of the Universe [2, 10]. And lastly, a (large) nonminimal coupling of the SM Higgs field with gravity leads to inflation consistent with the cosmological observations [4].

In the ν MSM, the dark matter sterile neutrino is produced at temperatures ~ 100 MeV due to the mixing with active leptonic flavors. The spectrum and the number density of produced particles depends essentially on three parameters: the mixing angle with ordinary neutrino, the sterile neutrino mass and the lepton asymmetry of the Universe (ΔL) at the production time (for a recent analysis, see [11, 12]; earlier considerations can be found in [6–9]). The comparison of theoretical predictions with cosmological and astrophysical observations (such as Ly- α data and x-ray observations, constraining the free streaming length of DM particles and their mixing angle with neutrinos, respectively) lead to the conclusion that the (low-temperature) lepton asymmetry must be much larger than the baryon asymmetry $\Delta B \sim 10^{-10}$, $\frac{\Delta L}{\Delta B} \geq 3 \times 10^5$.¹

As was shown in [3], the presence of a pair of nearly degenerate heavier neutral leptons in the ν MSM may lead to production of the requisite large lepton asymmetry below the electroweak scale without a conflict with observed small baryon asymmetry (generated by the same particles and by sphalerons at electroweak temperatures). Basically, the resonant production of ΔL occurs at decoupling or during decays of singlet fermions which are taking place well below the sphaleron freeze-out.

The requirement of generation of sufficient lepton asymmetry leads to the stringent conditions on the parameters of the model, partially analyzed in [3]. The most important of them is the level of the degeneracy of the pair of neutral leptons, which demands a severe fine-tuning of the masses and couplings of the ν MSM.

The aim of this paper is to study the stability of necessary fine-tuning against radiative corrections (only the tree-level analysis has been made in [3]). In particular, we will study the renormalization group (RG) evolution for the mass difference of the singlet fermions and formulate the conditions that can lead to a technically natural realization of the low-energy degeneracy, required for the low-temperature resonant leptogenesis, essential for DM production.

The paper is organized as follows: In Sec II we review the basic structure of the ν MSM and its parametrization. In Sec III we explain the necessity of the degeneracy between neutral leptons. In Sec IV we describe and analyze the renormalization group evolution of the essential parameters of the model. In Sec V we revisit different possible scenarios for singlet fermion mass splitting and in Sec VI we discuss an extension of the ν MSM by higher-dimensional operators. In Sec VII we discuss the phenomenological bounds obtained as a result of the fine-tuning. Finally, in Sec VIII, we summarize our results.

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¹ A possible way to evade this requirement is some modification of the ν MSM, allowing the DM sterile neutrino interactions with other new particles, see, e.g. [13–16]. Yet another possibility is related to primordial Higgs-inflation [17].

II. THE ν MSM AND ITS PARAMETRIZATION

We use the Lagrangian of the ν MSM in the following parametrization [3, 18]:

$$\mathcal{L}_{\nu MSM} = \mathcal{L}_0 + \Delta\mathcal{L}, \quad (1)$$

$$\begin{aligned} \mathcal{L}_0 = \mathcal{L}_{SM} + \sum_{I=2,3} \overline{N}_I i \partial_\mu \gamma^\mu N_I - (h_{\alpha 2} \overline{L}_\alpha N_2 \tilde{\phi} \\ + M \overline{N}_2^c N_3 + h.c.), \end{aligned} \quad (2)$$

$$\Delta\mathcal{L} = -h_{\alpha 3} \overline{L}_\alpha N_3 \tilde{\phi} - \frac{\Delta M}{2} \sum_{I=2,3} \overline{N}_I^c N_I + h.c., \quad (3)$$

where N_I are the right handed singlet leptons ($I = 2, 3$), ϕ and L_α ($\alpha = e, \mu, \tau$) are the Higgs and the lepton doublets, respectively, h is a matrix of the Yukawa coupling constants, M is the common mass of the two heavy neutral fermions, ΔM is the diagonal element of the Majorana mass matrix, and $\tilde{\phi}_i = \epsilon_{ij} \phi_j^*$, M and ΔM are taken to be real. We have omitted the DM sterile neutrino N_1 from the Lagrangian as its influence on the problem we are interested in is negligibly small (see [3] for details).

As has been shown in [3], one can solve for the active neutrino masses explicitly:

$$m = \left\{ 0, \frac{v^2}{M} [F_2 F_3 \pm |h^\dagger h|_{23}] \right\}, \quad (4)$$

where $F_i^2 \equiv [h^\dagger h]_{ii}$ and $v = 174$ GeV is the vacuum expectation value of the Higgs field. One encounters two different cases, namely the “normal hierarchy,” $m_1 = 0$, $m_2 = m_{sol} \simeq 0.009$ eV, $m_3 = m_{atm} \simeq 0.05$ eV, and the “inverted hierarchy,” $m_1 \approx m_2 \approx m_{atm} \simeq 0.05$ eV, $m_3 = 0$, $|m_1 - m_2| \approx m_{sol}^2 / (2m_{atm}) \simeq 8 \times 10^{-4}$ eV. Normal hierarchy corresponds to the case when $|h^\dagger h|_{23} \approx F_2 F_3$, and the inverted hierarchy to the case when $|h^\dagger h|_{23} \ll F_2 F_3$. Here m_{sol} , m_{atm} are the solar and atmospheric neutrino mass differences (for a review see [19]),

$$m_{sol}^2 = 7.65_{-0.20}^{+0.23} \times 10^{-5} \text{ eV}^2, \quad (5)$$

$$m_{atm}^2 = 2.40_{-0.11}^{+0.12} \times 10^{-3} \text{ eV}^2. \quad (6)$$

For the lepton and baryon asymmetries of the Universe, one of the essential parameters is the ratio

$$\epsilon^2 \equiv \frac{(h^\dagger h)_{33}}{(h^\dagger h)_{22}}, \quad (7)$$

measuring the relative strength of the coupling of the neutral leptons $N_{2,3}$ to the active flavors. Without loss of generality it can always be chosen in the region $\epsilon < 1$.

The most important parameter is the mass difference between the two heaviest neutrinos. Successful baryogenesis and leptogenesis necessitates the mass difference to

be small. At the tree level, there are two contributions to the mass difference: one related to the Majorana mass matrix and the other to the Higgs vacuum expectation value and Yukawa couplings [3]:

$$\delta m_{tree} = \frac{|m^2|}{M}, \quad (8)$$

where

$$m^2 \equiv 2(h^\dagger h)_{23} v^2 + 2M \Delta M. \quad (9)$$

Note that $2|h^\dagger h|_{23} v^2 = M|\Delta m_\nu|$, where Δm_ν is the difference between active neutrino masses, and $\Delta m_\nu \simeq 0.04$ eV (8×10^{-4} eV) for normal (inverted) hierarchy (Δm_ν by definition is a physical quantity and is the RG invariant).

The analysis of radiative corrections to Eq. (9) and its RG evolution plays a central role in our considerations (see also accompanying paper [20]).

III. THE FINE-TUNINGS NECESSARY FOR LOW-TEMPERATURE LEPTOGENESIS

It has been shown in [3] that the singlet fermions $N_{2,3}$ enter into thermal equilibrium at some temperature $T_+ \sim M_W$ (M_W being the intermediate vector boson mass) and freeze out later at $T_- \gtrsim M$. Then they decay at $T_d \lesssim M$. The low-temperature lepton asymmetry can be generated at $T \sim T_-$ and at $T \sim T_d$, to satisfy Sakharov out of equilibrium conditions.

The Yukawa coupling constants in the ν MSM are very small,

$$F_2 F_3 = \frac{\sum m_\nu M}{v^2} \simeq 8 \times 10^{-16} \kappa \frac{M}{\text{GeV}}, \quad (10)$$

where $\kappa = 1$ ($\kappa = 2$) for normal (inverted) hierarchy. Therefore, the production of substantial lepton asymmetry can only be possible in case of resonant $N_2 \leftrightarrow N_3$ transitions. This means that the oscillation rate Γ_{osc} should be of the same order as the scattering rate

$$\Gamma_s \simeq \frac{5G_F^2 T^5 m_{atm}}{\epsilon M}, \quad (11)$$

if the leptogenesis occurs at $T \sim T_-$, or the decay rate

$$\Gamma_N \simeq \frac{10G_F^2 M^4 m_{atm}}{192\pi^3 \epsilon}, \quad (12)$$

if $T \sim T_d$.

The quantity Γ_{osc} is related to the difference of physical masses of Majorana fermions δm_{phys} as $\Gamma_{osc} \sim \delta m_{phys}$, if $T < M$, or $\Gamma_{osc} \sim M \delta m_{phys} / T$ for $T > M$. So, to get a substantial lepton asymmetry at $T \sim T_-$, we have to require that

$$\frac{M \delta m_{phys}}{T_-} = \frac{T_-^2}{M_0}, \quad (13)$$

where $M_0 \approx M_{Pl}/1.66\sqrt{g_{eff}}$, $M_{Pl} = 1.2 \times 10^{19}$ GeV and the temperature dependence of the effective number of massless degrees of freedom g_{eff} may be taken from [11]. From (13), for $T \sim M \sim 1 - 10$ GeV we get,

$$\delta m_{phys}(T_-) \lesssim 10^{-18} - 10^{-16} \text{ GeV} . \quad (14)$$

An even stronger condition must be true for asymmetry generation in N decays. Taking again $M \sim 1 - 10$ GeV we have

$$\delta m_{phys}(T_d) \lesssim 10^{-23} - 10^{-19} \text{ GeV} . \quad (15)$$

The physical mass difference of singlet fermions in Eqs. (14,15) should be taken at the corresponding temperatures, T_- or T_d .

It is the smallness of these numbers in comparison with the observed neutrino mass difference Δm_ν (at least 8×10^{-13} GeV for the inverted hierarchy case) that leads to the fine-tuning problem. Indeed, there are several terms of different nature which make up the physical mass difference: the tree level Higgs contribution $\sim \Delta m_\nu$, the tree level Majorana contribution $\sim \Delta M$, the zero-temperature loop corrections, and finite-temperature corrections. The latter effects were analyzed in detail in [3], Sec. 5.1. It was found there that these corrections lead to the induced finite-temperature mass difference δm_{phys} which is of the order of the Hubble rate H at temperature T_- (see Eq. (7.32) of [3]) or smaller than H at T_d . In other words, they do not spoil the resonant character of low-temperature leptogenesis, provided the zero-temperature mass difference is tuned to small values, given in Eqs. (14,15) above. Therefore, we will concentrate on zero-temperature contributions in what follows. A part of the zero-temperature loop corrections can be absorbed into the tree-level Higgs contribution, converting it to the physical mass difference of active neutrinos [3]:

$$\delta m_{phys} = |\Delta m_\nu e^{i\alpha} + \Delta M| + \mathcal{O}\left(\frac{\Delta m_\nu}{16\pi^2} \frac{M^2}{v^2}\right) , \quad (16)$$

where α is some phase. Numerically, the last term in (16) is of the order of 10^{-18} GeV (for $M \sim 1$ GeV), which is of the same order as (14) and is much larger than (15). Therefore, the tuning of the tree-level Majorana contributions to the *physical value* Δm_ν must be done together with radiative corrections in order to achieve the required degeneracy $\delta m_{phys} \ll \Delta m_\nu$.

Yet another attitude can be used in discussing the fine-tuning. Suppose that the compensation of the *tree* contributions in (8) is associated with some symmetry which may potentially exist at some high energy scale, such as the Planck mass M_{Pl} . Then the physical mass-difference δm_{phys} , which is determined by the *low-energy* parameters, is not in general zero, due to the running of these parameters. The consideration of the running of the relevant parameters, together with computation of the radiative corrections [20], would allow then to estimate the "natural" values of the mass difference. This is the purpose of the next section.

IV. RG EVOLUTION OF THE MASS-DIFFERENCE

The RG running of the Majorana masses and of sterile-active Yukawa couplings can be extracted from [21, 22]

$$(4\pi)^2 \frac{d}{dt} M_R = (h^\dagger h) M_R + M_R (h^\dagger h)^T , \quad (17)$$

$$(4\pi)^2 \frac{d}{dt} h = \left\{ \frac{3}{2} h h^\dagger - \frac{3}{2} Y_e^\dagger Y_e - \frac{3}{4} g_1^2 - \frac{9}{4} g_2^2 + \text{Tr}[3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + h h^\dagger + Y_e^\dagger Y_e] \right\} h , \quad (18)$$

where $t \equiv \ln \frac{\mu}{\mu_0}$ (for definiteness we take $\mu_0 = \text{top quark mass}$), M_R is the Majorana mass matrix, Y_e is the diagonal charged lepton matrix, $Y_{u(d)}$ is the Yukawa coupling matrix for the up (down) quarks, and g_1, g_2 represent the gauge coupling for U(1) and SU(2), respectively. Since (9) contains the vacuum expectation value of the Higgs field, $v^2 = m_H^2/2\lambda$ (m_H^2 is the mass parameter in the SM, λ is the scalar self-coupling)², we will also need the RG runnings of m_H^2 and λ . They are given, for example, in [23]:

$$(4\pi)^2 \frac{dm_H^2}{dt} = m_H^2 \left(6\lambda - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 + 6y_t^2 \right) , \quad (19)$$

$$(4\pi)^2 \frac{d\lambda}{dt} = (12\lambda - 3g_1^2 - 9g_2^2)\lambda + \frac{3}{4} g_1^4 + \frac{3}{2} g_1^2 g_2^2 + \frac{9}{4} g_2^4 + 12(\lambda - y_t^2)y_t^2 . \quad (20)$$

To complete the system, we add the RG equation for the top quark Yukawa coupling y_t ,

$$(4\pi)^2 \frac{dy_t}{dt} = y_t \left(\frac{9}{2} y_t^2 - \frac{17}{12} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 \right) . \quad (21)$$

The RG evolution of other parameters of the SM will not be needed in this section.

Using the fact that $h \ll g_{1,2}$, and neglecting numerically small Yukawa couplings of charged leptons and quarks (except y_t), we get the RG equation for the Majorana mass difference and for h :

$$(4\pi)^2 \frac{d}{dt} \Delta M = 2M |h^\dagger h|_{23} , \quad (22)$$

$$(4\pi)^2 \frac{d}{dt} h = h \left\{ 3y_t^2 - \frac{3}{4} g_1^2 - \frac{9}{4} g_2^2 \right\} . \quad (23)$$

² In this paper, we use the Higgs potential form as:
 $V(\phi) = -\mu^2(\phi^\dagger \phi) + \frac{\lambda}{2}(\phi^\dagger \phi)^2$.

We consider the Yukawa couplings for the sterile neutrinos to be real in this section. This simplifies the computations, but does not change the numerical estimates and qualitative conclusions.

So, the change in ΔM due to RG evolution from the Planck scale to μ_0 is of the order of

$$\frac{\Delta m_\nu}{(4\pi)^2} \ln \left(\frac{M_{Pl}}{\mu_0} \right) \frac{M^2}{v^2} \simeq (6.5 \times 10^{-18} - 4 \times 10^{-16}) \frac{M^2}{\text{GeV}}, \quad (24)$$

depending on the type of neutrino hierarchy. This number for the inverted hierarchy is roughly of the same order as (14), but exceeds considerably (15).

The RG evolution of the Higgs contribution to the mass difference is much more substantial. In order to make an estimate of this contribution, we fix the initial condition for the Yukawa couplings at the Planck scale and run them down to the electroweak scale. For definiteness we take the inverted hierarchy case, the conclusions for the normal hierarchy are the same.

It was shown in [3] that for inverted hierarchy the coupling of N_2 to τ flavour (consideration of other leptonic families do not change the results) is

$$|h_{\tau 2}| = \frac{F_2}{2} \left| \cos \theta_{12} e^{-i\zeta} + i \sin \theta_{12} e^{i\zeta} \right|, \quad (25)$$

where $\tan^2 \theta_{12} \simeq 0.48$ from active neutrino oscillation data [19] and ζ is a phase factor that varies between 0 and 2π . A little algebra shows that

$$\left| \frac{h_{\tau 2}}{F_2} \right|_{\min} = 0.14, \quad \left| \frac{h_{\tau 2}}{F_2} \right|_{\max} = 0.7. \quad (26)$$

Similar relations hold for $|h_{\tau 3}|$ with F_2 replaced by F_3 .

Using (26) and (10), it follows that the maximum and minimum values of the Yukawa couplings, for $M = 1$ GeV and $\epsilon = 1$, are

$$|h_{\tau 2}|_{\min} \sim 3.4 \times 10^{-9}, \quad |h_{\tau 2}|_{\max} \sim 1.7 \times 10^{-8}. \quad (27)$$

A similar set of maximum and minimum values hold for $|h_{\tau 3}|$. The values of Yukawa couplings scale as $h_{\alpha 2} \propto \sqrt{M/\epsilon}$, $h_{\alpha 3} \propto \sqrt{M\epsilon}$.

Using the above-mentioned bounds along with Eqs. (19),(20) and (23), we obtain the RG evolution of the Higgs contribution down to the electroweak scale. Performing the necessary numerical computation, we find that the Higgs contribution to the mass-difference ranges from 10^{-10} GeV to 10^{-11} GeV depending on whether the maximum or minimum value is chosen for the Yukawa couplings. This value is of the order of Δm_ν , meaning that if m^2 in (9) is tuned to zero (implying $\delta m_{tree} = 0$) at the Planck scale, the physical mass difference between N_2 and N_3 will generically be of the order of Δm_ν , exceeding considerably the required values (14,15). A way out is a fine-tuning of the Higgs mass, which can be chosen in such a way that $\delta m_{tree}(M_{pl}) = \delta m_{tree}(M_W) = 0$. In Fig. 1, we give the RG evolution of the mass-difference $\delta m_{tree}(\mu)$ for a particular choice of Higgs mass (145 GeV), which leads to this situation.

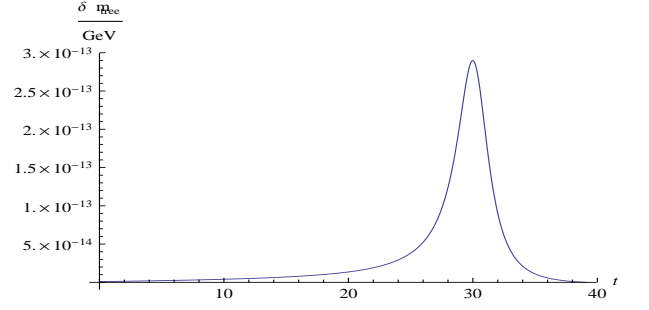


FIG. 1. RG evolution of δm_{tree} for a Higgs mass of $\simeq 145$ GeV.

V. SCENARIOS FOR SINGLET FERMION MASS DIFFERENCE REVISITED

To summarize, if there were a compensation in the tree-level contribution to the mass-difference δm_{tree} due to existence of some symmetry at a higher-energy scale (M_{pl}), it does not exist at the electroweak scale and the physical mass difference acquires a nonzero value, which is of the same order as that of the change in Higgs contribution: $10^{-10} - 10^{-11}$ GeV. The reason is that the Majorana contribution to m^2 runs much more slowly than the Higgs one. One can arrive to (14,15) either by tuning of the Higgs mass, or by tuning the initial condition for ΔM at the Planck scale. In Sec. VI we will discuss how this situation can be changed in some extension of the ν MSM. Meanwhile, we will describe the possible scenarios (sf. [3]) for singlet fermion mass difference accounting for RG behavior studied in Sec. IV. Depending on the relative importance of the different contributions to δm_{phys} , they may be classified as follows:

- **Scenario Ia:** $\Delta M(M_{pl}) = 0$. In this case the physical mass difference is mostly due to the tree-level Higgs condensate and loop corrections. One can easily check from Eqs. (4), (9) that this leads to

$$\delta m_{phys} \approx \Delta m_\nu. \quad (28)$$

Thus, $\delta m_{phys} \simeq m_{atm} - m_{sol} \approx 5 \times 10^{-11}$ GeV for normal hierarchy and $\delta m_{phys} \simeq \frac{\Delta m_{sol}^2}{2m_{atm}} \approx 8 \times 10^{-13}$ GeV for inverted.

- **Scenario Ib:** This corresponds to the situation in which the tree-level Higgs and the Majorana contribution are of the same order of magnitude at the Planck scale, including the case when there is a compensation of the two contributions, $m^2(M_{pl}) = 0$. In this case and without any special fine-tuning one gets

$$\delta m_{phys} \sim \Delta m_\nu. \quad (29)$$

- **Scenario II:** The *physical* mass difference of the singlet fermions is much smaller than the active

neutrino mass difference, i.e.

$$\delta m_{phys} \ll \Delta m_\nu. \quad (30)$$

Only this scenario can lead to production of substantial low-temperature lepton asymmetry and thus to resonant sterile neutrino dark matter production. It requires a fine-tuning between contributions of the different nature (Higgs, Majorana, and loop corrections) and thus is “unnatural” in a technical sense³.

- **Scenario III:** The Majorana contribution dominates, in which case

$$\delta m_{phys} \gg \Delta m_\nu. \quad (31)$$

The scenarios **Ia**, **Ib**, and **III** are natural in the technical sense. However, they do not lead to low-temperature lepton asymmetry. In Sec. VI we will present an extension of the ν MSM in which **Scenario II** can be realized as a natural possibility.

VI. HIGHER-DIMENSIONAL OPERATORS

The ν MSM can be extended in several ways, which may break the relations (4,8,9), being the basis of the analysis of the previous sections. The simplest possibility, in the spirit of effective field theories, is to add higher-dimensional operators. There are five independent five-dimensional operators which can be constructed from the fields of the ν MSM. One contains the fields of the SM only,

$$c_1^{\alpha\beta} \overline{L_\alpha} \tilde{\phi} \phi^\dagger L_\beta^c. \quad (32)$$

The other two include the singlet fermions as follows:

$$c_2^{IJ} \overline{N_I^c} N_J \phi^\dagger \phi, \quad c_3^{IJ} (\partial_\mu \overline{N_I})^c \partial_\mu N_J. \quad (33)$$

Yet another two include L and N simultaneously,

$$c_4^{\alpha I} \overline{\not{D} L_\alpha} N_I^c \tilde{\phi}, \quad c_5^{\alpha I} \overline{L_\alpha} (\not{D} N_I)^c \tilde{\phi}. \quad (34)$$

Here c_i are new coupling constants with dimension GeV^{-1} . The operator (32) can change the relation (4), whereas operators (33,34) can change (8,9).

There are several possible ways how the **Scenario II** can be made technically natural. The first one is based on the use of the operator (32), which contributes to the active fermion mass difference. The idea is as follows. Suppose that at M_{pl} the following two relations hold simultaneously:

$$\Delta M = 0, \quad (h^\dagger h)_{23} = 0. \quad (35)$$

These initial conditions require that the active neutrino mass hierarchy must be inverted, as follows from Eq. (4). Then, if charged lepton Yukawa couplings Y_e are set to zero, the Lagrangian of the ν MSM has an extra global leptonic $U(1)$ symmetry [18]. This means, that the relations (35) are RG scale-independent, and that the singlet fermions will remain exactly degenerate. In fact, the charged leptonic Yukawa couplings violate explicitly this symmetry and thus lead to the breaking of exact degeneracy at small energies. So, some mass difference between the singlet fermions will be generated. As we will see below, it is much smaller than the observed mass difference between active neutrinos. The inclusion of the operator (32) is needed to generate the observed active neutrino mass difference and requires $c_1 \simeq 1/(10^{16} \text{GeV})$; the common mass of active neutrinos is due to $N_{2,3}$.

Let us make an estimate of the splitting between $N_{2,3}$ owing to charged lepton Yukawas. Among Y_e the tau-lepton coupling is the largest, and $Y_e^\dagger Y_e$, which was previously omitted from Eq. (18), has the form

$$Y_e^\dagger Y_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau^2 \end{pmatrix}. \quad (36)$$

The evolution of ΔM and $(h^\dagger h)_{23}$, following from Eqs. (17,18) with initial conditions (35) is shown in Figs. 2 and 3 respectively. We give only the plots corresponding to the minimum value of the singlet fermion Yukawa couplings [see (27)] and for $M = 1 \text{ GeV}$. The energy

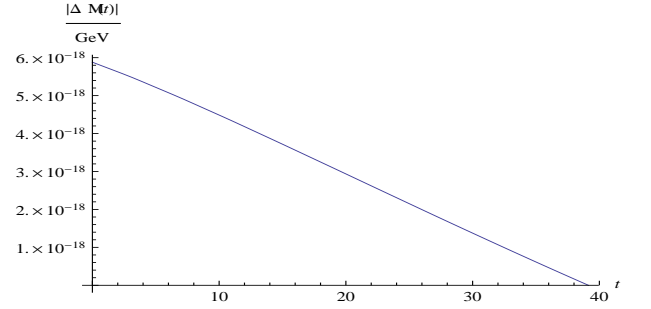


FIG. 2. RG evolution of ΔM .

scale has been varied from the top quark mass ($t = 0$) to Planck scale ($t = 39.14$). We can see that both $|\Delta M|$ and $(h^\dagger h)_{23}$ decrease steadily with the increase of t .

Numerically, for the minimal value of $h_{\tau 2}$ [**case (a)** for future reference], the mass difference is given by (sf eq. (16))

$$\delta m_{phys} \simeq \left| 5 \times 10^{-17} e^{i\alpha} - 1.2 \times 10^{-17} \frac{M^2}{\text{GeV}^2} \right| \text{ GeV}, \quad (37)$$

establishing a typical scale $\sim 10^{-17} \text{ GeV}$.

Analogous computation for the maximal value of $h_{\tau 2}$ [**case (b)**] leads us to the similar dependence: the mass-difference is given by

$$\delta m_{phys} \simeq \left| 1.3 \times 10^{-15} e^{i\alpha} - 3 \times 10^{-16} \frac{M^2}{\text{GeV}^2} \right| \text{ GeV}, \quad (38)$$

³ By “technically” natural we mean the situation in which the fine-tuning made at the high energy scale persists to small energies.

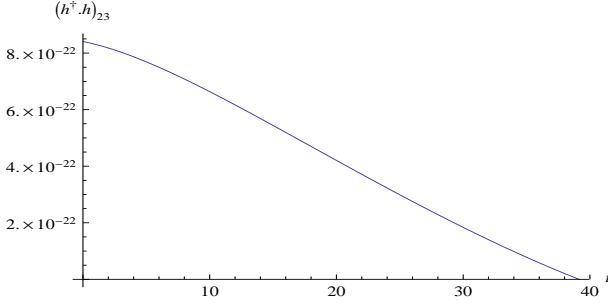


FIG. 3. RG evolution of $|h^\dagger h|_{23}$.

with a typical scale of $\sim 10^{-16}$ GeV.

In order to find the different possible values of mass-difference, we analyze the two cases separately for different values of M .

- **case (a):** From Eq. (37) we see that for a particular choice of the Majorana mass ($M \simeq 2$ GeV), the mass difference can be made arbitrarily small by a suitable choice of the Majorana phase (α). In particular, the mass difference vanishes for $\alpha = 0$. Therefore, it may be safely concluded that for $M \simeq 2$ GeV, the following bounds hold true for δm_{phys} :

$$0 \leq \delta m_{phys} \leq 10^{-16} \text{ GeV}. \quad (39)$$

We now turn to the two other possibilities: $M \ll 2$ GeV and $M \gg 2$ GeV. Considering $M \ll 2$ GeV first, we can see that

$$\delta m_{phys} = 5 \times 10^{-17} \text{ GeV}, \quad (40)$$

while for $M \gg 2$ GeV,

$$\delta m_{phys} = 1.2 \times 10^{-17} \frac{M^2}{\text{GeV}}. \quad (41)$$

- **Case (b):** Performing similar analysis, we see that once again for $M \simeq 2$ GeV, the mass difference becomes arbitrarily small, depending on the choice of the Majorana phase. We thus establish the bound for δm_{phys} in this case:

$$0 \leq \delta m_{phys} \leq 2.6 \times 10^{-15} \text{ GeV}. \quad (42)$$

For $M \ll 2$ GeV,

$$\delta m_{phys} = 1.3 \times 10^{-15} \text{ GeV}, \quad (43)$$

while for $M \gg 2$ GeV,

$$\delta m_{phys} = 3 \times 10^{-16} \frac{M^2}{\text{GeV}}. \quad (44)$$

Thus, we see that the mass difference δm_{phys} so obtained in the two cases is indeed much smaller than the active neutrino mass difference, what is needed for the

low-temperature resonant leptogenesis. As we have already said, the role of the higher-dimensional operator (32) is to provide the additional contribution to mass difference of the active flavors. The amplitude of all other operators must be small enough in order not to spoil the relations (37,38), which is technically natural. In Sec. VII we will consider different bounds on parameters of the ν MSM which appear in this scenario.

Yet another possible way to make the **Scenario II** technically natural is based on operators (34). These operators contribute to the singlet fermion mass with the terms of the order $c_4^{\alpha I} h_{\alpha J} v^2$, $c_3^{\alpha I} h_{\alpha J} v^2$. If the RG running of some combination of these contributions happens to be the same as the running of the Higgs contribution $2(h^\dagger h)_{23} v^2 / M$, this can be used to cancel the largest effect (24). If true, the physical mass difference can be “naturally” made of the order of the last term in Eq. (16). The analysis of this possibility goes beyond the scope of the present paper.

VII. PHENOMENOLOGICAL BOUNDS ON PARAMETERS OF ν MSM FROM NATURALNESS

Let us assume that the physical mass difference is indeed given by the relations derived in Sec. VI. Is the baryogenesis due to singlet fermion oscillations still operational? What can be said about the parameters ϵ and M from the requirement of resonant dark matter production?

To answer the first question, let us determine the crucial parameter of the baryogenesis – the number of oscillations $x(T_{\text{sph}})$ of singlet fermions before the freezing of sphalerons at temperature $T_{\text{sph}} \sim 150$ GeV [3]. The high-temperature mass difference of the singlet fermions comes from two-loop graphs which include the square of charged lepton Yukawa coupling and square of $h_{\alpha I}$ and corresponds to the double Higgs exchange. It can be estimated as

$$\Delta M(T)^2 \sim \left(\frac{h_\tau y_\tau}{16} \right)^2 T^2, \quad (45)$$

leading to

$$x(T_{\text{sph}}) \simeq \left(\frac{h_\tau y_\tau}{16} \right)^2 \frac{M_0}{T_{\text{sph}}}. \quad (46)$$

Yet another contribution to $\Delta M(T)^2$ comes from the Higgs condensate. It is of the same order of magnitude as (45), since at the sphaleron freeze out $v(T) \sim T$. Numerically, $x(T_{\text{sph}}) \sim (10^{-7} - 10^{-6})/\epsilon$. Since baryon asymmetry is proportional to x for $x \ll 1$, it is automatically smaller than the low-temperature lepton asymmetry by a factor $\sim 10^6$ (for $\epsilon \sim 1$), providing a potential explanation of this hierarchy. At the same time, with this value of x , the baryon asymmetry can be as large as $\Delta B \sim 10^{-9}$,

exceeding the observed one. In other words, the answer to the first question is positive.

To analyze the second question, we will consider two choices of $h_{\tau 2}$, referred to as **(case (a))** and **(case (b))** above. As has been shown in [3], the lepton asymmetry can be generated in either by decays or oscillations of the sterile neutrinos. We will consider these two situations separately.

A. Leptogenesis by decay of sterile neutrinos

Considering the case of decays first, it has been shown in [3] that the maximal lepton asymmetry Δ which can be generated in decays of $N_{2,3}$ is of the order

$$\Delta \approx \Delta_{\max} \frac{\epsilon M^2}{M_0 \delta m_{\text{phys}}}, \quad (47)$$

where $\Delta_{\max} = 2/11$. The condition that the decays of $N_{2,3}$ occur above the dark matter creation temperature ~ 100 MeV reads

$$\frac{M}{\text{GeV}} > 1.4 \left(\frac{\epsilon}{2 \times 10^{-3}} \right)^{\frac{1}{4}}. \quad (48)$$

To produce a necessary amount of dark matter sterile neutrinos, it is required that [12]

$$\Delta \geq 2 \times 10^{-3}. \quad (49)$$

- **Case (a):** When $M \ll 2$ GeV, using (40), (47) and (48), we arrive at the minimum value of M to be 3 GeV, which contradicts the starting assumption that $M \ll 2$ GeV. On the other hand, for $M \gg 2$ GeV, using (41) and (47), we arrive at a lower bound on ϵ :

$$\epsilon \gtrsim 0.1. \quad (50)$$

Corresponding minimum value of M [from (48)] is ~ 4 GeV.

- **Case (b):** Performing a similar analysis as before, we once again reject the case when $M \ll 2$ GeV. For the case when $M \gg 2$ GeV, we obtain a lower bound on ϵ to be 2.3, conflicting with the condition $\epsilon < 1$.

To summarize, a sufficient lepton asymmetry in decays of $N_{2,3}$ can be generated in **case (a)** for $M \gtrsim 4$ GeV and $\epsilon \gtrsim 0.1$. Or, one has to require the fine-tuning $M \simeq 2$ GeV and $\alpha \ll 1$, making the physical mass difference even smaller than $\sim 10^{-16}$ GeV.

B. Leptogenesis from coherent oscillations of sterile neutrinos

As has been shown in [3], the most important parameter which determines the value of lepton asymmetry generated at temperature T_- is the number of oscillations

$x(T_-)/2\pi$ of $N_{2,3}$ given by

$$x(T_-) = \frac{0.15\kappa B}{\epsilon} (G_F M_0)^2 m_{\text{atm}} \delta m_{\text{phys}}(T), \quad (51)$$

where κ is 1 or 2 depending on normal or inverted hierarchy, $B = 5$ and G_F is the Fermi coupling constant. The case $\delta m_{\text{phys}}(0) = 0$, in which $\delta m_{\text{phys}}(T_-) \neq 0$ due to finite-temperature effects, leads to $x(T_-) \equiv x_T \simeq 10$ and to some region in the ϵ, M plane leading to the required lepton asymmetry (49). This analysis stays in force also if $\delta m_{\text{phys}}(0) \neq 0$, provided

$$\delta m_{\text{phys}}(0) < \frac{x_T \epsilon}{0.15\kappa B (G_F M_0)^2 m_{\text{atm}}}. \quad (52)$$

We will give below the corresponding parts of the phase space. For this end we find $x(T_-)$ from Eq. (51) replacing $\delta m_{\text{phys}}(T)$ by δm_{phys} defined in Eqs. (39-44).

- **Case (a):** For $M \ll 2$ GeV, using (40) and (51), we get

$$x(T_-) = \frac{0.25}{\epsilon}, \quad (53)$$

which implies $\epsilon \geq 0.025$ to satisfy inequality $x(T_-) < x_T$. For $M \gg 2$ GeV, using (41) and (51), we get

$$x(T_-) = \frac{0.06}{\epsilon} \frac{M^2}{\text{GeV}^2}, \quad (54)$$

which implies $\epsilon \geq 0.006 M^2 / \text{GeV}^2$. Plotting ϵ as a function of M , for the two situations, the allowed values of $\epsilon - M$ are depicted by the shaded portion in Fig. 4.

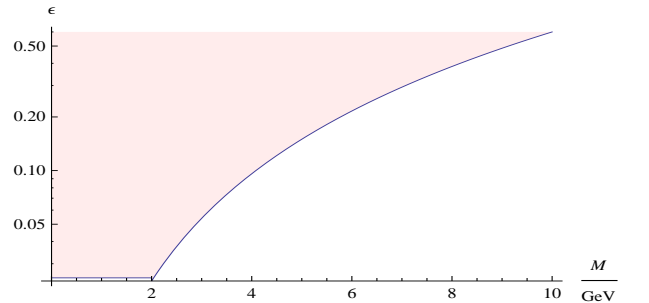


FIG. 4. Shaded region of the plot represents the values of $\epsilon - M$ in which the analysis of Ref. [3] is not changed for the **case (a)**. The ϵ axis is shown in logarithmic scale.

- **Case (b):** Similar analysis for $M \ll 2$ GeV leads to

$$x(T_-) = \frac{6.4}{\epsilon}, \quad (55)$$

which implies $\epsilon \geq 0.64$. Again for $m \gg 2$ GeV, we arrive at the relation:

$$x(T_-) = \frac{1.48}{\epsilon} \frac{M^2}{\text{GeV}^2}. \quad (56)$$

which implies $\epsilon \geq 0.148M^2/\text{GeV}^2$. These regions are shown in Fig. 5.

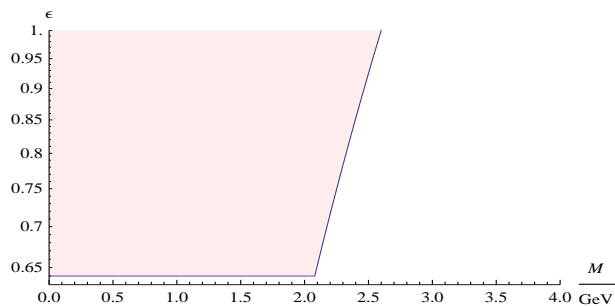


FIG. 5. The same as in Fig. 4 for the **case (b)**. In this plot, we only show the region where $\epsilon \leq 1$, which we assume to be true throughout the paper.

As in Sec. VII A, the case when $M \simeq 2 \text{ GeV}$ and $\alpha \ll 1$ is special, and all values for ϵ and δm_{phys} , found in Sec. VI are allowed.

VIII. CONCLUSIONS

The generation of lepton asymmetry below the sphaleron freeze-out temperature enables generation of a large lepton asymmetry without leaving a trace on the much smaller baryon asymmetry of the Universe. The production of dark matter sterile neutrinos is dependent on the lepton asymmetry present at the time of produc-

tion. In this paper we studied the RG evolution of the mass splitting between neutral leptons $N_{2,3}$ of the νMSM , essential for resonant production of low-temperature lepton asymmetry, followed by resonant production of dark matter sterile neutrinos. We found that the mass differences of the order or greater than the observed mass differences in the active neutrino sector are natural in the technical sense. In other words, the RG running of it from the Planck to the low-energy scale leads to corrections $\sim \Delta m_\nu$.

At the same time, the low-temperature resonant leptogenesis requires the splitting that is much smaller than $\sim \Delta m_\nu$ and thus is “fine-tuned.” We described an extension of the νMSM by higher-dimensional operators, in which this fine-tuning is due to an approximate symmetry of the theory at the Planck scale and which is not spoiled by the RG evolution. It requires the hierarchy of neutrino masses to be inverted. We analyzed the constraints on the masses and couplings of the singlet leptons in this scenario and demonstrated its feasibility.

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- [1] T. Asaka, S. Blanchet, and M. Shaposhnikov, Phys. Lett. **B631**, 151 (2005), arXiv:hep-ph/0503065.
 - [2] T. Asaka and M. Shaposhnikov, Phys. Lett. **B620**, 17 (2005), arXiv:hep-ph/0505013.
 - [3] M. Shaposhnikov, JHEP **08**, 008 (2008), arXiv:0804.4542 [hep-ph].
 - [4] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. **B659**, 703 (2008), arXiv:0710.3755 [hep-th].
 - [5] A. Boyarsky, O. Ruchayskiy, and M. Shaposhnikov, Ann. Rev. Nucl. Part. Sci. **59**, 191 (2009), arXiv:0901.0011 [hep-ph].
 - [6] S. Dodelson and L. M. Widrow, Phys. Rev. Lett. **72**, 17 (1994), arXiv:hep-ph/9303287.
 - [7] X.-D. Shi and G. M. Fuller, Phys. Rev. Lett. **82**, 2832 (1999), arXiv:astro-ph/9810076.
 - [8] A. D. Dolgov and S. H. Hansen, Astropart. Phys. **16**, 339 (2002), arXiv:hep-ph/0009083.
 - [9] K. Abazajian, G. M. Fuller, and M. Patel, Phys. Rev. D **D64**, 023501 (2001), arXiv:astro-ph/0101524.
 - [10] E. K. Akhmedov, V. A. Rubakov, and A. Y. Smirnov, Phys. Rev. Lett. **81**, 1359 (1998), arXiv:hep-ph/9803255.
 - [11] T. Asaka, M. Laine, and M. Shaposhnikov, JHEP **01**, 091 (2007), arXiv:hep-ph/0612182.
 - [12] M. Laine and M. Shaposhnikov, JCAP **0806**, 031 (2008), arXiv:0804.4543 [hep-ph].
 - [13] M. Shaposhnikov and I. Tkachev, Phys. Lett. **B639**, 414 (2006), arXiv:hep-ph/0604236.
 - [14] A. Kusenko, Phys. Rev. Lett. **97**, 241301 (2006), arXiv:hep-ph/0609081.
 - [15] A. Anisimov, Y. Bartocci, and F. L. Bezrukov, Phys. Lett. **B671**, 211 (2009), arXiv:0809.1097 [hep-ph].
 - [16] F. Bezrukov and D. Gorbunov, JHEP **05**, 010 (2010), arXiv:0912.0390 [hep-ph].
 - [17] F. Bezrukov, D. Gorbunov, and M. Shaposhnikov, JCAP **0906**, 029 (2009), arXiv:0812.3622 [hep-ph].
 - [18] M. Shaposhnikov, Nucl. Phys. **B763**, 49 (2007), arXiv:hep-ph/0605047.
 - [19] A. Strumia and F. Vissani(2006), arXiv:hep-ph/0606054.
 - [20] A. Roy(2010), arXiv:1006.4007 [hep-ph].
 - [21] S. Antusch, J. Kersten, M. Lindner, and M. Ratz, Phys. Lett. **B538**, 87 (2002).
 - [22] Y. Lin, L. Merlo, and A. Paris(2009), arXiv:0911.3037 [hep-ph].
 - [23] A. J. Davies, S. Meljanac, and I. Picek, Phys. Lett. **B238**, 431 (1990).